INVESTMENT TRIVIA

The Not-so-Normal Distribution of Equity Returns

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DRW INVESTMENT RESEARCH

Part 5:

The Implications of Equity Return Distributions for Investors

Data:

- The daily closing values of the FTSE JSE All Share Index from 1 July 1995 to 31 March 2011.

- The closing values of the FTSE JSE All Share Index at month-end from 31 January 1960 to 31 March 2011.

Results:
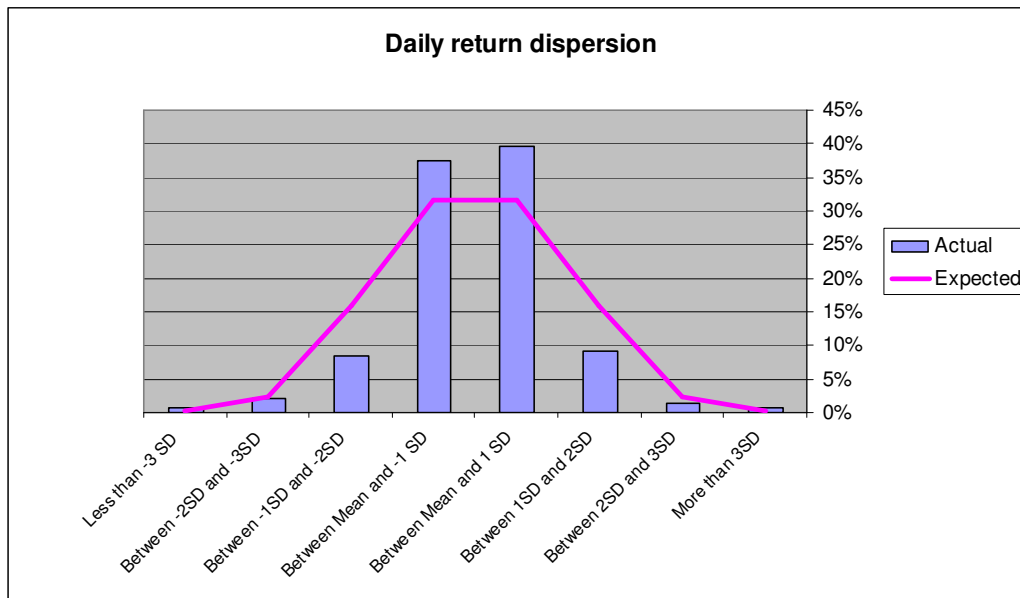
1.   Daily return distribution

Data: 1 July 1995 to 31 March 2011

Number of trading days: 3,923

Average return: 0.057%

Median return: 0.082%

Standard deviation (SD):   1.3%

Distribution of daily returns

Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	0.8%
Between 2SD and 3SD	2.28%	1.5%
Between 1SD and 2SD	15.87%	9.1%
Between Mean and 1 SD	31.7%	39.7%
Between Mean and -1 SD	31.7%	37.5%
Between -1SD and -2SD	15.87%	8.5%
Between -2SD and -3SD	2.28%	2.1%
Less than -3 SD	0.13%	0.8%

- Fat tails (leptokurtic distribution) – more-than-expected extreme positive and negative return observations.
- The distribution is negatively-skewed or skewed to the left.

2.   Monthly return distribution

Data: January 1960 to 31 March 2011
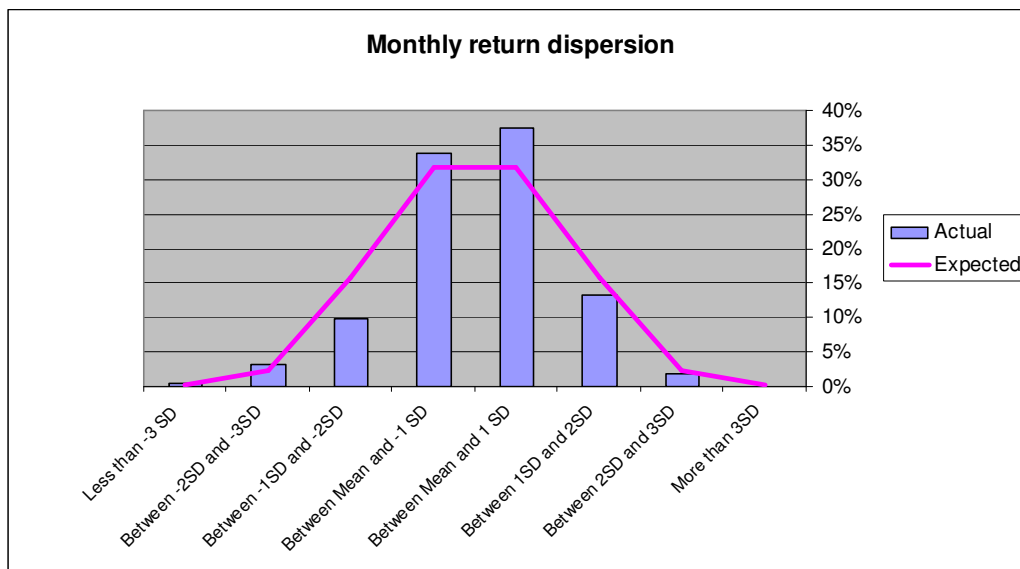
Number of months: 614

Average return: 1.2%

Median return: 1.4%

Standard deviation (SD):   6.2%

Distribution of quarterly returns

Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	0.0%
Between 2SD and 3SD	2.28%	1.8%
Between 1SD and 2SD	15.87%	13.2%
Between Mean and 1 SD	31.7%	37.5%
Between Mean and -1 SD	31.7%	33.9%
Between -1SD and -2SD	15.87%	9.9%
Between -2SD and -3SD	2.28%	3.3%
Less than -3 SD	0.13%	0.5%

- Fat tails (leptokurtic distribution) – more-than-expected extreme positive and negative return observations.
- The distribution is negatively-skewed or skewed to the left.

3.   Quarterly return distribution

Data: January 1960 to 31 March 2011

Number of quarters (no overlapping):   204

Average return: 3.7%

Median return: 4.3%

Standard deviation (SD):   11.8%

Distribution of quarterly returns

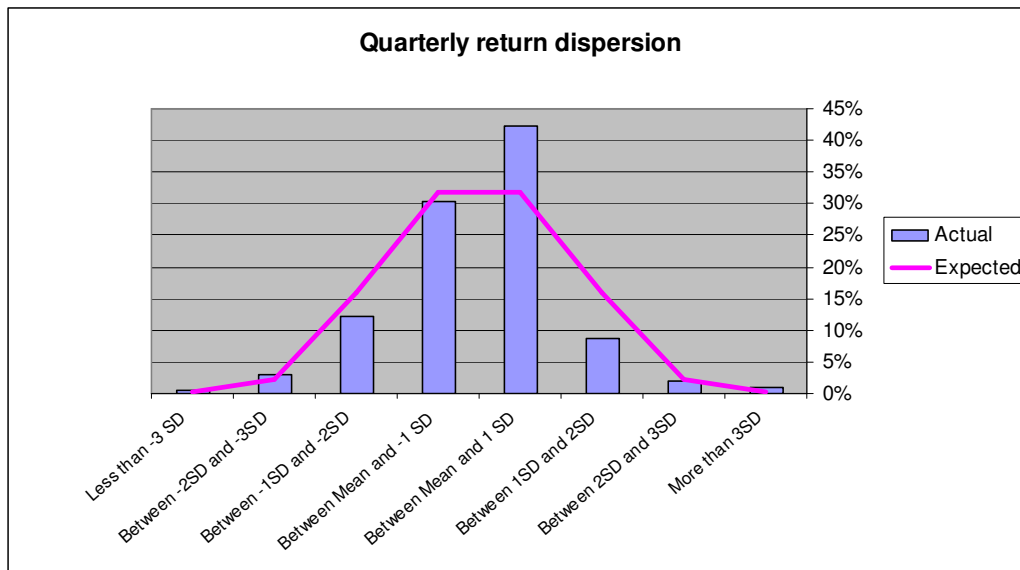| Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	1.0%
Between 2SD and 3SD	2.28%	2.0%
Between 1SD and 2SD	15.87%	8.8%
Between Mean and 1 SD	31.7%	42.2%
Between Mean and -1 SD	31.7%	30.4%
Between -1SD and -2SD	15.87%	12.3%
Between -2SD and -3SD	2.28%	2.9%
Less than -3 SD	0.13%	0.5%

- Fat tails (leptokurtic distribution) – more-than-expected extreme positive and negative return observations.
- The distribution is negatively-skewed or skewed to the left.

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4.   Bi-annual return distribution

Data: January 1960 to 31 March 2011

Number of periods (no overlapping):   101

Average return: 7.8%

Median return: 8.6%

Standard deviation (SD):   17.8%

Distribution of 6-monthly returns

| Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	1.0%
Between 2SD and 3SD	2.28%	1.0%
Between 1SD and 2SD	15.87%	6.9%
Between Mean and 1 SD	31.7%	46.5%
Between Mean and -1 SD	31.7%	31.7%
Between -1SD and -2SD	15.87%	9.9%
Between -2SD and -3SD	2.28%	3.0%
Less than -3 SD	0.13%	0.0%
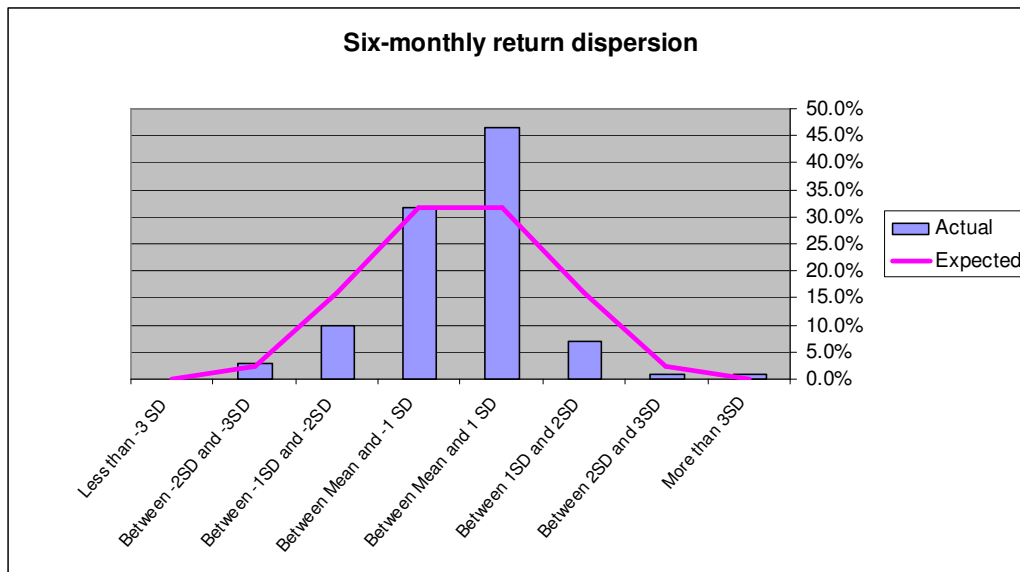
- Fat tails (leptokurtic distribution) – more-than-expected extreme positive return observations.
- The distribution is negatively-skewed or skewed to the left.

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5.   Annual return distribution (see Appendix 1 for additional information)

Data: 31 March 1960 to 31 March 2011

Number of periods (no overlapping):   51

Average return: 15.9%

Median return: 20.2%

Standard deviation (SD):   26.38%

Minimum return: -38.3%

Maximum return: 63.8%

Distribution of annual returns

| Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	0.0%
Between 2SD and 3SD	2.28%	0.0%
Between 1SD and 2SD	15.87%	13.7%
Between Mean and 1 SD	31.7%	39.2%
Between Mean and -1 SD	31.7%	25.5%
Between -1SD and -2SD	15.87%	19.6%
Between -2SD and -3SD	2.28%	2.0%
Less than -3 SD	0.13%	0.0%

Annual Returns
Period ended March
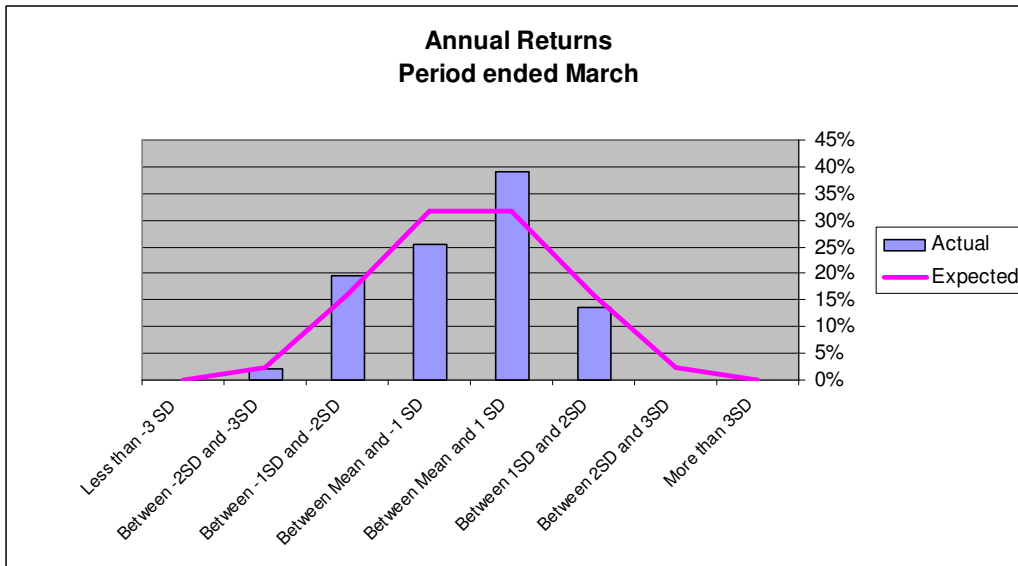
- Less extreme positive and negative return observations than predicted by normal distribution.
- The distribution is negatively-skewed or skewed to the left.

6.   Two-year return distribution (see Appendix 2 for additional information)

Data: 31 March 1960 to 31 March 2011

Number of periods (no overlapping):   25

Average return (annualised):   15.0%

Median return (annualised):   15.7%

Standard deviation (SD):   19.0%

Minimum return (annualised):   -28.5%

Maximum return (annualised):   44.6%

Distribution of two-year returns

| Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	0.0%
Between 2SD and 3SD	2.28%	0.0%
Between 1SD and 2SD	15.87%	16.0%
Between Mean and 1 SD	31.7%	36.0%
Between Mean and -1 SD	31.7%	36.0%
Between -1SD and -2SD	15.87%	8.0%
Between -2SD and -3SD	2.28%	4.0%
Less than -3 SD	0.13%	0.0%

Two-year returns
Period ended March
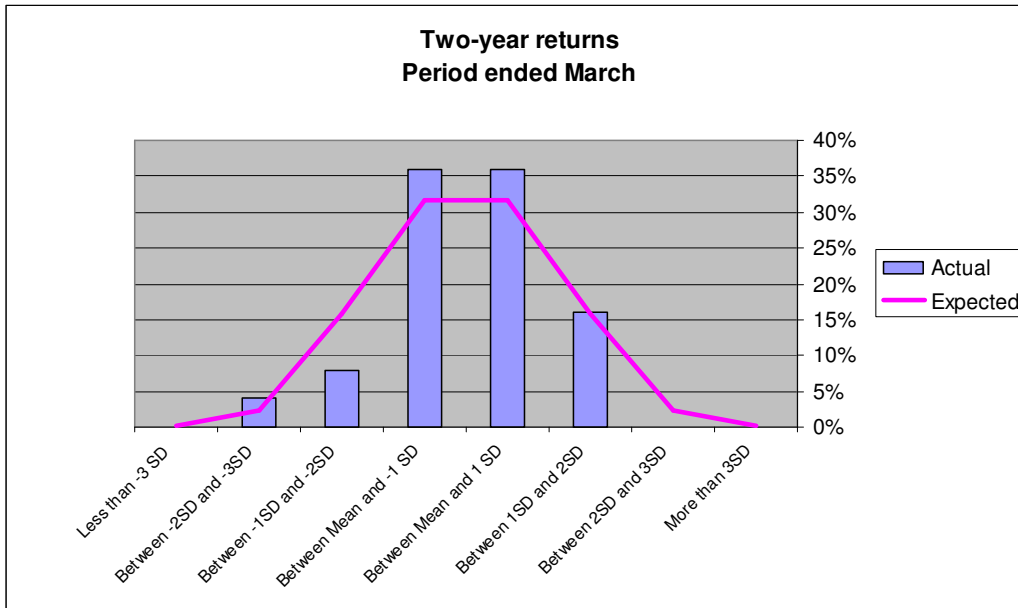
- Less extreme negative return observations than predicted by normal distribution, generally much more extreme positive return observations than negative returns.
- The distribution is negatively-skewed or skewed to the left.

7. Three-year return distribution (see Appendix 3 for additional information)

Data: 31 March 1960 to 31 March 2011

Number of periods (no overlapping):   17

Average return (annualised):   13.7%

Median return (annualised):   11.4%

Standard deviation (SD):   14.9%

Minimum return (annualised):   -15.5%

Maximum return (annualised):   44.3%

Distribution of three-year returns

Deviation from mean	Normal distribution expectations	Actual observations
More than 3SD	0.13%	0.0%
Between 2SD and 3SD	2.28%	5.9%
Between 1SD and 2SD	15.87%	11.8%
Between Mean and 1 SD	31.7%	29.4%
Between Mean and -1 SD	31.7%	41.2%
Between -1SD and -2SD	15.87%	11.8%
Between -2SD and -3SD	2.28%	0.0%
Less than -3 SD	0.13%	0.0%

Three-year returns
Period ended March
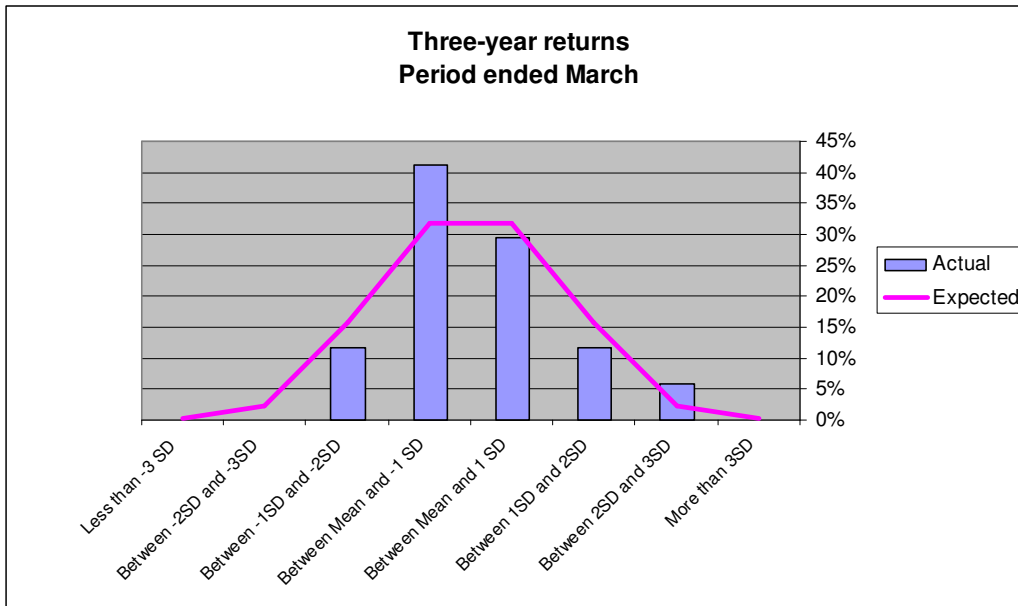
- Basically all extreme observations to the right (positive), negative return observations much less than predicted by normal distribution.
- The distribution is positively-skewed or skewed to the right.

Implications for investors:

- We tend to underestimate the extent or extremeness of negative returns over short-term intervals, but overestimate the occurrence of negative returns over longer holding periods. Or, we underestimate the short-term risk of equity investing, but overestimate the risk of long-term equity investing.

- Alternatively, we incorrectly extrapolate the short-term experiences of equity investing in formulating an opinion about the riskiness of long-term equity investing.

- Risk modelling tools, such as Monte Carlo simulations that use the normal distribution as a core return distribution assumption tend to simulate a too pessimistic return scenario for long-term investors, i.e. real-life experiences should be better than predicted for long-term investors.

- These risk models, however, underestimate the extremeness of price movements that is crucial for the viability and sustainability of high-risk investment strategies. Typically, leveraging and derivative instruments are used in such strategies to amplify security positions and bets. Unlike long-term investors, these investors (speculators) need to be right all the time and cannot afford large (temporary) losses at a specific point in time. In all likelihood, extreme short-term losses will result in permanent losses for such investors (speculators).

APPENDIX 1

Annual returns: Distributions by month

Month ended	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of periods	51	51	51	50	50	50	50	50	50	50	50	50
Average return	14.9%	15.3%	15.9%	16.6%	16.2%	16.1%	15.9%	16.1%	15.7%	15.6%	15.3%	15.4%
Median	13.8%	18.9%	20.2%	18.0%	17.6%	12.2%	12.3%	11.7%	16.9%	15.1%	13.4%	13.5%
Std Dev	24.3%	25.3%	26.3%	28.3%	27.2%	28.3%	26.3%	27.5%	25.9%	25.4%	23.9%	23.9%
Min	-26.4%	-39.8%	-38.3%	-39.6%	-49.4%	-47.8%	-39.4%	-36.2%	-30.9%	-33.0%	-34.8%	-29.7%
Max	65.9%	75.0%	63.8%	73.0%	93.1%	111.3%	102.8%	95.0%	84.2%	86.0%	74.3%	82.3%

Actual observations

Range	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
More than 3SD	0.0%	0.0%	0.0%	0.0%	0.0%	2.0%	2.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Between 2SD and 3SD	2.0%	2.0%	0.0%	0.0%	2.0%	2.0%	2.0%	2.0%	2.0%	4.0%	4.0%	4.0%
Between 1SD and 2SD	17.6%	15.7%	13.7%	14.0%	10.0%	8.0%	8.0%	16.0%	14.0%	12.0%	12.0%	12.0%
Between Mean and 1 SD	29.4%	33.3%	39.2%	38.0%	40.0%	34.0%	36.0%	26.0%	36.0%	30.0%	32.0%	30.0%
Between Mean and -1 SD	31.4%	31.4%	25.5%	30.0%	34.0%	42.0%	40.0%	44.0%	32.0%	40.0%	36.0%	38.0%
Between -1SD and -2SD	19.6%	15.7%	19.6%	18.0%	12.0%	10.0%	10.0%	12.0%	16.0%	14.0%	14.0%	16.0%
Between -2SD and -3SD	0.0%	2.0%	2.0%	0.0%	2.0%	2.0%	2.0%	0.0%	0.0%	0.0%	2.0%	0.0%
Less than -3 SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

APPENDIX 2

Two-year returns: Distributions by month

Month ended	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of periods	25	25	25	24	24	24	24	24	24	24	24	24
Average return	14.4%	14.7%	15.0%	15.0%	14.9%	14.4%	14.4%	14.4%	14.3%	14.2%	14.0%	14.0%
Median	15.1%	16.0%	15.7%	14.9%	14.4%	13.7%	13.3%	12.0%	10.7%	12.4%	14.4%	13.7%
Std Dev	17.8%	18.0%	19.0%	20.6%	19.7%	18.6%	17.8%	17.5%	17.4%	16.2%	14.1%	14.1%
Min	-22.6%	-23.8%	-28.5%	-30.2%	-28.9%	-23.6%	-20.5%	-21.7%	-23.4%	-23.4%	-21.1%	-14.5%
Max	49.8%	49.4%	44.6%	49.8%	47.7%	43.8%	56.7%	52.5%	50.0%	45.1%	48.3%	52.5%

Actual observations

Range	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
More than 3SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Between 2SD and 3SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.2%	4.2%	4.2%	0.0%	4.2%	4.2%
Between 1SD and 2SD	16.0%	12.0%	16.0%	16.7%	20.8%	16.7%	16.7%	16.7%	12.5%	12.5%	8.3%	4.2%
Between Mean and 1 SD	40.0%	40.0%	36.0%	33.3%	29.2%	33.3%	20.8%	25.0%	20.8%	37.5%	37.5%	41.7%
Between Mean and -1 SD	28.0%	32.0%	36.0%	33.3%	37.5%	37.5%	45.8%	37.5%	50.0%	37.5%	41.7%	29.2%
Between -1SD and -2SD	12.0%	12.0%	8.0%	12.5%	8.3%	8.3%	12.5%	12.5%	8.3%	8.3%	4.2%	16.7%
Between -2SD and -3SD	4.0%	4.0%	4.0%	4.2%	4.2%	4.2%	0.0%	4.2%	4.2%	4.2%	4.2%	4.2%
Less than -3 SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

APPENDIX 3

Three-year returns: Distribution by month

Month ended	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of periods	17	17	17	16	16	16	16	16	16	16	16	16
Average return	13.1%	13.4%	13.7%	14.8%	14.7%	14.3%	14.1%	14.0%	13.5%	12.9%	12.9%	13.0%
Median	14.1%	13.7%	11.4%	12.0%	12.6%	12.4%	12.2%	12.5%	11.7%	11.6%	11.3%	12.1%
Std Dev	14.1%	14.7%	14.9%	15.7%	14.5%	12.0%	11.6%	11.7%	12.0%	8.9%	8.8%	8.7%
Min	-13.3%	-14.0%	-15.5%	-16.6%	-13.8%	-9.4%	-5.0%	-1.8%	-3.0%	-2.0%	-2.3%	-3.6%
Max	37.4%	41.3%	44.3%	43.7%	41.2%	32.3%	41.7%	40.2%	40.5%	32.5%	35.1%	30.7%

Actual observations

Range	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
More than 3SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Between 2SD and 3SD	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	6.3%	6.3%	6.3%	6.3%	6.3%	6.3%
Between 1SD and 2SD	17.6%	11.8%	11.8%	18.8%	25.0%	25.0%	6.3%	6.3%	6.3%	6.3%	6.3%	12.5%
Between Mean and 1 SD	35.3%	41.2%	29.4%	25.0%	18.8%	25.0%	31.3%	31.3%	25.0%	18.8%	31.3%	25.0%
Between Mean and -1 SD	29.4%	29.4%	41.2%	43.8%	37.5%	37.5%	43.8%	43.8%	43.8%	50.0%	43.8%	43.8%
Between -1SD and -2SD	17.6%	17.6%	11.8%	12.5%	18.8%	12.5%	12.5%	12.5%	18.8%	18.8%	12.5%	12.5%
Between -2SD and -3SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Less than -3 SD	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

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