Applying Index Investing Strategies:

Optimising Risk-adjusted Returns

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For the untrained eye the ensuing topic might appear highly theoretical, academic and even intimidating, yet it contains a very powerful message of how index investing can fit into an overall investment strategy, which I shall endeavor to convey in the simplest terms possible.

The active manager invariably deviates from the index benchmark and constructs a portfolio that does not replicate the benchmark. For example, a manager will typically include larger portions of small-cap stocks in the portfolio than its respective weight in the market index. Normally the manager's portfolio would be more equally-weighted than those of the index. Furthermore, investment constraints may prohibit a manager to accumulate more holdings of a stock than its mandate will allow (maximum 10% holding of one stock in a portfolio), which is especially relevant in the South African context with its skewed market characteristics (dominant mining and resources sectors).

The rationale for risk-adjusted performance measures rests upon the fact that, if an active manager pursues a low-risk investment strategy, one should not expect the same returns from that strategy compared with a manager that follows high-risk strategies. A manager's skill could then only be judged upon the return that was delivered versus the risk taken to deliver that return.

A similar argument could be put forward in comparing active and index investment strategies. If, for example, index investing outperformed active investing over time it could have been achieved with a relatively higher risk profile than active investing and consequently on a risk-adjusted basis would show equal or lesser qualities. Therefore, a meaningful comparison between the two strategies is not possible without adjusting for risk.

First, a basic understanding of some investment risk terminology is required:

Risk-free return: The return that you will achieve from a cash investment – default (capital) risk is for all practical purposes zero.

Excess return: An investment portfolio's return less the risk-free return. For example if an actively-managed portfolio delivered 14% versus a cash alternative of 7%, your excess return would have been 7%. At the same time a market index investment achieved 15% over the same period, thus an excess return of 8%.

Volatility:	The standard deviation (deviation from the mean) of excess returns – say the actively-managed portfolio's volatility was 15% and the index's volatility was 18%.

Sharpe ratio:	A widely-used risk metric in the investment industry. The ratio expresses the excess return of your portfolio against its volatility, given by:

$$\frac{\text{(Portfolio return – risk-free return)}}{\text{Volatility of Portfolio}}$$

In terms of the above ratio the actively-managed portfolio outperformed the market portfolio (0.47 versus 0.44). Also, the Sharpe ratio implies the probability that an investment will outperform the risk-free rate of return (assuming a normal distribution). Thus, the higher the Sharpe ratio of an investment, the better is the chance to outperform cash.

Beta:	The percentage change expected in your portfolio's excess return per unit change in market excess return, given by:

$$\frac{\text{Covariance between managed portfolio return and market return}}{\text{Variance of market return}}$$

Say in this example the *Beta* is 80%; what it actually means is that for every one percent change in market return, the actively-managed portfolio will change 0.80% – thus the portfolio value is expected to increase at a slower rate than the market index portfolio, but at the same time to decline at a slower pace as well.

Alpha:	The out-performance of the portfolio's excess return over the market excess return, given by a regression function:

$$r_p - r_f = \alpha_p + \beta_p(r_m - r_f)] \quad \text{, and therefore,}$$
$$\alpha_p = r_p - [r_f + \beta_p(r_m - r_f)]$$

Where:

α_p =	the abnormal excess return of the actively-managed portfolio over the market excess return;
r_p =	actively-managed portfolio's return over period (14%);
r_f =	risk-free return over period (7%);
β_p =	beta of the actively-managed portfolio's return with market return (80%);
$(r_m - r_f)$ =	market excess return over period (8%)

The alpha of the actively-managed portfolio is thus 0.60% (14%-13.4%) – which is magic, this is what all active managers are striving for! The actively-managed portfolio, despite its slightly lower nominal return than the market index return (14% versus 15%), outperformed the index on a risk-adjusted basis.

Obviously, you are not always going to be this lucky, as in the above example, with all active managers – the majority of them under-perform the market in the long run, even on a risk-adjusted basis and thus have negative alphas (note that the market portfolio cannot exhibit any alpha, only active managers can).

Say you want to implement this methodology (recommended) and you want to evaluate and compare five different actively-managed funds, which all exhibit positive alphas over a reasonable period. Will you simply pick the one with the highest alpha over this period? No, unfortunately that might be the wrong or unsuitable choice, because that manager might take very large bets against the market index, which will imply that whenever this manager is making his/her calls correctly, he/she will score big time, but then the opposite holds as well.

The trick is to measure the manager's alpha attained against the risk that cannot be explained by the market (index) portfolio – known as the *non-systematic risk*, *idiosyncratic risk* or *tracking error*. Hereby the alpha of the manager that is taking large bets against the market index (large tracking error) can be measured against a manager's alpha that is following a more moderate approach betting against the market. This comparative measure is known as the information ratio (IR) and can be formularised as:

Information ratio: Alpha of fund
                                      Tracking error of fund

                                      And,

Tracking error: Part of the total volatility of fund return that cannot be explained by market or systematic risk, where systematic risk is explained by the coefficient of determination (r^2), hence non-systematic risk is explained by:

$$\sigma(e_P) = \sqrt{(1-r^2)(\sigma^2_P)}$$

The *information ratio* is thus an appropriate tool to identify those active funds that delivered the highest out-performance (alpha) with the least tracking error. It focuses on the active return (above the expected market return) versus the active risk taken, which could have been diversified away by holding a portfolio similar to the market.

The rational choice would be to select those active funds with the highest information ratios, but bear in mind these ratios are probably not constant, since out-performance against the market index is not constant. Therefore, using a strategy of only active management might go sour, and might it be worthwhile to build into your investment plan some "safety net" precautions, which I shall illustrate in the following sections of this article.

For now we have enough tools at hand to do a proper analysis of investment fund performance and to judge whether such funds should form part of the overall investment strategy. Therefore, I am going to present now an example of where one has five different funds to choose from, and by using some optimising techniques an ideal combination between index and active investing can be found, as well as which active funds to select as part of your active investing strategy.

In table 1 you are presented with the performance records of five different actively-managed funds, together with the market index and cash returns, over a ten-year investment period. Table 2 exhibits an analysis of the respective performances. From these a decision must be made which active funds to select and how much of each should be selected.

Table1: Historical performance records of five actively-managed funds and a market portfolio (index)

Years	Fund A	Fund B	Fund C	Fund D	Fund E	Market	Cash
1	16%	17%	15%	8%	15%	14%	10%
2	-12%	-15%	-12%	-3%	-14%	-10%	10%
3	22%	23%	26%	19%	14%	22%	7%
4	-5%	-12%	-8%	-10%	-17%	-12%	7%
5	18%	27%	23%	45%	51%	35%	5%
6	27%	30%	37%	24%	29%	25%	6%
7	-16%	-5%	-8%	-3%	-11%	-9%	7%
8	30%	21%	25%	19%	38%	27%	8%
9	15%	9%	9%	8%	16%	17%	7%
10	15%	17%	9%	17%	7%	5%	7%

Table 2: Analysis of Returns

Metric	Fund A	Fund B	Fund C	Fund D	Fund E	Market	Cash
Initial Investment	100,000	100,000	100,000	100,000	100,000	100,000	100,000
End Value	256,052	260,532	270,094	294,241	277,880	263,909	203,997
Yield (annualized)	9.86%	10.05%	10.45%	11.40%	10.76%	10.19%	7.39%
Average Return	11.00%	11.20%	11.60%	12.40%	12.80%	11.40%	7.40%
Average Excess Return	3.60%	3.80%	4.20%	5.00%	5.40%	4.00%	0.00%
Std Dev (Excess)	16.70%	17.07%	17.40%	17.00%	23.30%	17.71%	
Variance (Excess)	2.79%	2.92%	3.03%	2.89%	5.43%	3.14%	
Beta	85%	90%	91%	87%	128%	100%	
Alpha	0.18%	0.21%	0.56%	1.52%	0.29%	0.00%	
R squared	0.91	0.93	0.93	0.91	0.97	1.00	
Sharpe	0.216	0.223	0.241	0.294	0.232	0.226	
Prob(z>Cash)	58.53%	58.81%	59.54%	61.56%	59.17%	58.93%	
Tracking error	5.12%	4.51%	4.72%	5.21%	3.90%		
Information Ratio	0.04	0.05	0.12	0.29	0.07		

From table 2: All five actively-managed funds display positive alpha and information ratios. Also, note that the end-values for funds A and B are lower than the market portfolio, but since it is less risky than the market portfolio, positive alpha and information ratios are attained. It is all about reward-to-risk, and not just absolute values.

Let us assume none of these five active funds will under-perform the market portfolio on a risk-adjusted basis. Furthermore, we are not worried about possible tracking errors, in other words we do not put any limit on the tracking error of the investment portfolio. Since all five active funds exhibit positive alphas (outperforming the index on a risk-adjusted basis) and will continue to do so, we are not likely to select an index investment, our only concern is to decide which of the five active funds to select. How do we go about it?

Should we just pick Fund D (by far the highest information ratio) or should we pick the highest two, or perhaps split the investment between the five different funds (equal allocation), or is there a more scientific approach available?

A methodology was developed by Jack Treynor and Fischer Black, called the Treynor-Black model (TB), which focuses specifically on identifying those funds with the highest alphas, but at the same time minimising the tracking error from the market portfolio, and thus maximizing the information ratio. The methodology is basically showing the "smoothest" path to some decent return.

To illustrate this concept I am using the same performance data (as in table 2) to back-test the different approaches. Following the TB-methodology the optimal combination and returns are shown in table 3. Furthermore, the results of an equal allocation (20% allocated to each active fund) and the case where if we had perfect foresight that active fund D would deliver the highest risk-adjusted return are shown.

Table 3: Combining different active funds, no limit on tracking error

Metric	Market Portfolio	Optimal (TB)	Equal	Perfect Selection
Allocation A		6%	20%	0%
Allocation B		9%	20%	0%
Allocation C		21%	20%	0%
Allocation D		48%	20%	100%
Allocation E		16%	20%	0%
Total Active Weight	0%	100%	100%	100%
Total Index Weight	100%	0%	0%	0%
Final Value from R100,000	263,909	283,973	274,745	294,241
Yield	10.19%	11.00%	10.64%	11.40%
Average Return	11.40%	12.10%	11.80%	12.40%
Average Excess Return	4.00%	4.70%	4.40%	5.00%
Std Dev (Excess)	17.71%	17.26%	17.42%	17.00%
Variance (Excess)	3.14%	2.98%	3.04%	2.89%
Beta	100%	94.54%	96.18%	86.97%
Alpha	0%	0.92%	0.55%	1.52%
R squared	100%	97.03%	97.79%	90.60%
Sharpe	0.226	0.272	0.253	0.294
Prob(Z>Cash)	58.96%	60.76%	59.99%	61.59%
Tracking error	0%	2.97%	2.59%	5.21%
Information Ratio		0.3097	0.2133	0.2919

The highest possible absolute return would have been if you had perfect foresight in selecting fund D, but it has a high tracking error, resulting in that the TB solution have a higher information ratio.

As noted earlier it is unrealistic to assume that funds exhibiting high tracking errors will significantly outperform the market every year, especially when measured over a longer investment period. The chances are real that you may severely under-perform the market portfolio. What if some portfolio limits are placed on the maximum tracking error – is it possible in such case to enhance the information ratio any further?

Table 4 depicts such a scenario where the maximum tracking error is limited to 2%. In this case one is "forced" to include an index fund in your portfolio, since the index fund will not have any tracking error (assume) and the different combinations have higher tracking errors than the maximum of 2%. Furthermore, the same allocations are used for the different methods as in table 3.

Table 4: Combining different active funds, 2% tracking error limit

Metric	Market Portfolio	Optimal (TB)	Equal	Perfect Selection
Allocation A		6%	20%	0%
Allocation B		9%	20%	0%
Allocation C		21%	20%	0%
Allocation D		48%	20%	100%
Allocation E		16%	20%	0%
Total Active Weight	0%	71%	95%	38%
Total Index Weight	100%	29%	5%	62%
Final Value from R100,000	263,909	278,475	274,237	276,508
Yield	10.19%	10.78%	10.61%	10.71%
Average Return	11.40%	11.90%	11.78%	11.78%
Average Excess Return	4.00%	4.50%	4.38%	4.38%
Std Dev (Excess)	17.71%	17.28%	17.42%	17.05%
Variance (Excess)	3.14%	2.99%	3.03%	2.91%
Beta	100%	96.10%	96.37%	95.00%
Alpha	0%	0.66%	0.52%	0.58%
R squared	100%	98.50%	98.01%	98.68%
Sharpe	0.226	0.260	0.251	0.257
Prob(Z>Cash)	58.96%	60.30%	59.95%	60.17%
Tracking error	0%	2.12%	2.46%	1.96%
Information Ratio		0.3108	0.2134	0.2980

From table 4 it can be seen that in all three active allocation decisions it is possible to increase the IR of your investment portfolio by including an index strategy – in other words more reward-to-risk can be created by including an index fund in your overall investment plan.

The same rationale is applicable for enhanced index strategies and is used as their primary selling feature – slight tracking errors are allowed in the portfolio in order to generate alphas with resultant high information ratios.

By using this methodology an "efficient frontier" of various investment strategies (active and passive) can be constructed for a given level of active risk (tracking error) required. The optimised results of such a model are shown in table 5 and graphically illustrated in figure 1.

Table 5:   Optimal investment strategy allocations

Type Of Fund	0% Active Risk	0.5% Active Risk	1.0% Active Risk	1.5% Active Risk	2.0% Active Risk	2.5% Active Risk	3% Active Risk
Index Fund	100	72	44	16	0	0	0
Enhanced Index	0	16	33	50	52	39	17
Active Growth	0	5	10	15	21	26	35
Active Value	0	5	10	15	21	26	35
Active Concentrated	0	2	3	4	6	9	13

Source: Waring & Siegel, 2003
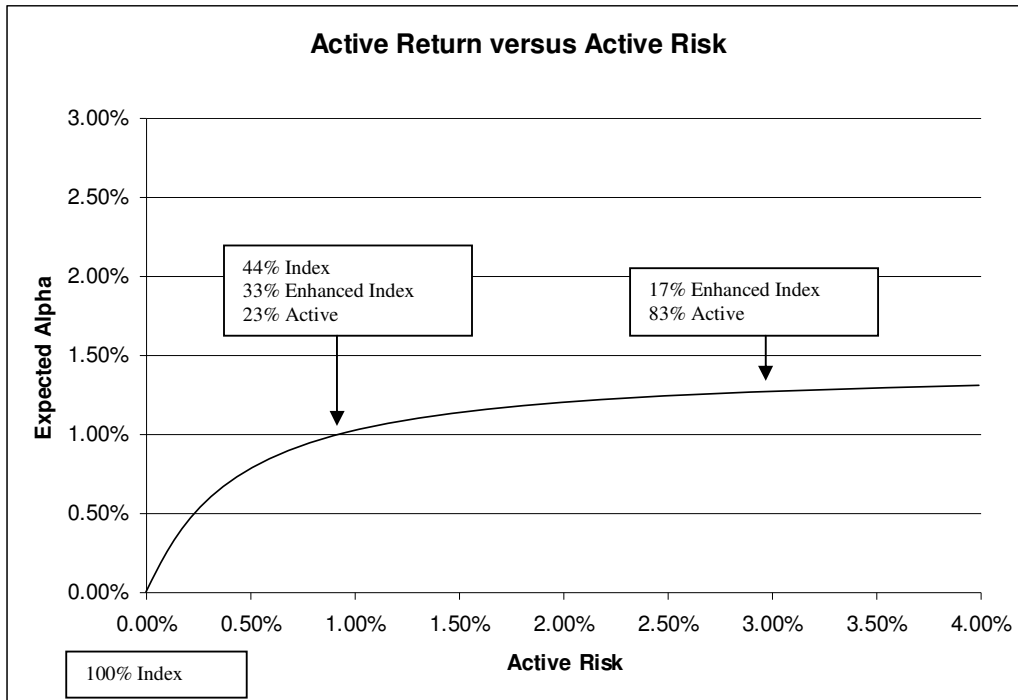
Figure 1:   **Efficient Frontier of Optimal Combination Strategies**

Source: Adapted from Waring & Siegel, 2003

Summary:

The purpose of this article is to illustrate how index investing can play a role in increasing the reward-to-risk of your overall investment plan. Investing should not be only about chasing absolute values all the time, which will in any event be uncertain until you eventually realise your investment, but also the paths you select to get to those returns.

By including index (enhanced index) strategies in your investment plan the tracking error of your investment is reduced, thus limiting the probability that your actual investment returns will be sub par compared with pure market returns.

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